

$I = 0, 1 \pi\pi$ and $I = 1/2 K\pi$ Scattering using Quark Born Diagrams

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Abstract

We extend the quark Born diagram formalism for hadron-hadron scattering to processes with valence $q\bar{q}$ annihilation, specifically $I = 0$ and $I = 1 \pi\pi$ and $I = 1/2 K\pi$ elastic scattering. This involves the s -channel hybrid annihilation process $q^2\bar{q}^2 \rightarrow q\bar{q}g \rightarrow q^2\bar{q}^2$ and conventional s -channel $q\bar{q}$ resonances, in addition to the t -channel gluon exchange treated previously using this formalism. The strength of the t -channel gluon amplitude is fixed by previous studies of $I = 2 \pi\pi$ and $I = 3/2 K\pi$ scattering. The s -channel resonances $\rho(770)$, $K^*(892)$, $f_0(1400)$ and $K_0^*(1430)$ are incorporated as

relativized Breit-Wigner amplitudes, with masses and energy-dependent widths fitted to experimental phase shifts. The strength of the s -channel gluon “hybrid” annihilation diagrams is problematical since the perturbative massless-gluon energy denominator must be modified to account for the effect of confinement on the energy of the virtual hybrid state. Our naive expectation is that near threshold the hybrid diagrams are comparable in magnitude but opposite in sign to the contribution predicted using massless perturbative gluons. Fitting the strength of this amplitude to $I = 0$ $\pi\pi$ and $I = 1/2$ $K\pi$ S-wave data gives a result consistent with this expectation. We find good agreement with experimental phase shifts from threshold to 0.9 GeV in $\pi\pi$ and 1.6 GeV in $K\pi$ using this approach. We conclude that the most important contribution to low energy S-wave scattering in these channels arises from the nonresonant quark Born diagrams, but that the low-energy wings of the broad s -channel resonances $f_0(1400)$ and $K_0^*(1430)$ also give important contributions near threshold. The nonresonant contributions are found to be much smaller in $L > 0$ partial waves, but may be observable in $I = 1$ $\pi\pi$ and $I = 1/2$ $K\pi$ P-wave phase shifts.

1. Introduction

The determination of hadron-hadron interactions in terms of quark and gluon degrees of freedom is an important goal of the nonrelativistic quark potential model. Historically these studies first concentrated on the nucleon-nucleon interaction, which is well determined experimentally and is fundamental to nuclear physics. Nonperturbative techniques such as the resonating group method [1] and variational approaches [2] lead to reasonable descriptions of the short- and intermediate-range nucleon-nucleon interaction in terms of quark forces. (For a review of this work on NN to 1989 see Shimizu [3].) Studies of other hadron-hadron channels in terms of quark and gluon interactions have also been successful. Weinstein and Isgur [4] carried out variational studies of pseudoscalar meson-meson interactions using the nonrelativistic quark potential model, and found weakly-bound deuteron-like “ $K\bar{K}$ molecules”, which have been identified with the $f_0(975)$ and $a_0(980)$. They also used their variational techniques to extract equivalent low-energy meson-meson potentials, which with some qualifications give reasonable results for $\pi\pi$ and $K\pi$ elastic phase shifts [5, 6]. The resonating group techniques have also been applied to the kaon-nucleon system [7] and give results similar to the observed $I = 0$ and $I = 1$ S-wave phase shifts. There may be some discrepancies, however, and the $I = 0$ KN phase shift in particular is not yet well determined experimentally and merits further study, which may be possible at DAPHNE [8].

Recently Barnes and Swanson [9, 10] introduced a perturbative Born-order formalism for hadron-hadron scattering in terms of quark and gluon degrees of freedom, based on a single interaction, usually one-gluon-exchange (OGE) followed by quark line interchange. This analytical technique reproduces many of the successes of the earlier nonperturbative calculations when applied to scattering processes which are free of valence quark annihilation. In some cases such as the $I = 2$ $\pi\pi$ and $I = 3/2$ $K\pi$ S-waves the Born diagrams are in remarkably good agreement with experiment from threshold to the maximum experimental invariant mass of about 1.5 GeV. The reactions studied to date using this method are $I = 2$ $\pi\pi$ [9, 10], $I = 3/2$ $K\pi$ [11], certain vector-vector channels [10, 12], $I = 0, 1$ KN [13] and NN , $N\Delta$ and $\Delta\Delta$ [14], and several other related channels. One conclusion of these studies was that the powerful but complicated nonperturbative techniques were unnecessary in some channels, notably $\pi\pi$ and $K\pi$, because the perturbative amplitude alone gives a good description of the data.

Motivated by the successes of this simple perturbative approach, in this paper we generalize this technique to scattering processes with valence annihilation. General hadron-hadron scattering amplitudes include contributions from resonance production, resonance exchange and $q\bar{q}$ annihilation (s -channel gluon exchange), in addition to the t -channel gluon exchange considered previously in this formalism. In principle each of these contributions may be important, so reactions in annihilation channels can be quite complicated. Here we consider pseudoscalar-pseudoscalar (Ps-Ps) elastic scattering with valence annihilation, and assume that the important amplitudes arise from t -channel gluon exchange, s -channel gluon exchange and s -channel resonance production. The first contribution is treated using the quark Born diagram techniques developed previously, and we find that the t -channel gluons which dominated $I = 2$ $\pi\pi$ and $I = 3/2$ $K\pi$ are numerically rather weak here. The second scattering mechanism, s -channel gluon exchange, is treated using the same quark Born techniques. Special care must be taken with this process, however, to incorporate the mass

of the intermediate hybrid state. This changes s -channel gluon exchange from a repulsive to an attractive interaction in the cases studied here, and makes the overall magnitude of this effect rather uncertain. Here we fit it to experimental S-wave phase shifts, and compare the fitted strength to our naive estimate based on an effective gluon mass of ≈ 1 GeV. The third contribution, from s -channel resonances, is incorporated phenomenologically by treating these as relativized Breit-Wigner phase shifts with free masses and widths, which we fit to the data.

In section 2 we describe our techniques in detail for $I = 0$ and $I = 1$ $\pi\pi$ scattering, and give numerical estimates of the contribution of each effect to the scattering length. In section 3 we give the corresponding results for $I = 1/2$ $K\pi$ scattering, which is formally similar to $\pi\pi$ but is somewhat more complicated due to the strange quark mass. In section 4 we carry out detailed fits to experimental data sets for $I = 0$ and $I = 1$ $\pi\pi$ and $I = 1/2$ $K\pi$ phase shifts and discuss the relative importance of resonant and nonresonant contributions. We conclude with a brief summary of our results and suggestions for future work.

2. Detailed Results for $I = 0$ and $I = 1$ $\pi\pi$ Scattering Amplitudes

We consider three contributions to hadron-hadron scattering in annihilation channels,

- 1) t -channel gluon exchange,
- 2) s -channel gluon exchange (valence $q\bar{q}$ annihilation), and
- 3) s -channel resonances.

A fourth contribution which is often discussed is t -channel meson exchange. We exclude this scattering mechanism in $\pi\pi$ and $K\pi$ scattering because the most important such contribution, one pion exchange, is not present in Ps-Ps scattering, and in any case the contribution of t -channel meson exchange to low-energy hadron-hadron scattering has probably been overestimated. For a discussion of this issue see [15].

Taking the three mechanisms in order, their contributions to the scattering amplitude are as follows:

- 1) *t -channel gluon exchange*

In earlier studies of Ps-Ps elastic scattering [4-6] it was found that the dominant scattering mechanism involves the OGE spin-spin hyperfine interaction, which between quarks i and j is

$$H_{ij} = -\frac{8\pi\alpha_s}{3m_i m_j} (\lambda_i^a/2) \cdot (\lambda_j^a/2) (\vec{S}_i \cdot \vec{S}_j) \delta(\vec{r}_{ij}) . \quad (1)$$

A similar conclusion was reached earlier for NN scattering [1-3]. An explanation of hyperfine dominance in Ps-Ps scattering was given by Barnes and Swanson [9, 10], who found that the matrix elements of $\lambda_i^a \cdot \lambda_j^a \vec{S}_i \cdot \vec{S}_j$ in the four Born-order quark scattering diagrams for $(q\bar{q})(q\bar{q}) \rightarrow (q\bar{q})(q\bar{q})$ all have the same sign in this channel. In contrast, the spin-independent matrix elements of $\lambda_i^a \cdot \lambda_j^a$ (which multiply the spin-independent color Coulomb and linear confining terms) interfere destructively between the diagrams. In the Ps-Ps annihilation

channels we consider here the same conclusions apply, so we again assume dominance of the spin-spin color hyperfine term in t -channel gluon exchange.

The t -channel gluon exchange contribution to $I = 0$ $\pi\pi$ scattering can be determined from the previous quark Born study of $I = 2$ $\pi\pi$ scattering [9]. The only difference is the flavor factor, which gives a Hamiltonian matrix element of

$$h_{fi}^{I=0}(t\text{-ch. gluon}) = -\frac{1}{2}h_{fi}^{I=2}(t\text{-ch. gluon}) = -\frac{4\pi\alpha_s}{9m_q^2}\frac{1}{(2\pi)^3}\left[\exp\left\{- (1-\mu)\frac{k^2}{4\beta_\pi^2}\right\} + \exp\left\{- (1+\mu)\frac{k^2}{4\beta_\pi^2}\right\} + \frac{16}{\sqrt{27}}\exp\left\{-\frac{k^2}{3\beta_\pi^2}\right\}\right]. \quad (2)$$

The $I = 1$ $\pi\pi$ matrix element from t -channel gluon exchange is zero; this result and the $I = 0/I = 2$ ratio both follow immediately from the observation that $\pi^+\pi^- \not\rightarrow \pi^+\pi^-$ through this mechanism. Here $\mu = \cos(\theta_{c.m.})$, where $\theta_{c.m.}$ is the center of the mass scattering angle, and k is the magnitude of the asymptotic three-momentum of each meson in the $c.m.$ frame. This is the matrix element of the spin-spin OGE term calculated between single-Gaussian $q\bar{q}$ wavefunctions, summed over all four permutations of gluon exchanges and quark line interchanges. The wavefunction parameter β_π is usually taken to be about 0.3 GeV in the nonrelativistic quark model. In [9] a fit to the $I = 2$ $\pi\pi$ S-wave phase shift gave $\beta_\pi = 0.337$ GeV, $\alpha_s = 0.6$ and $m_q = 0.33$ GeV, which we also use here. (Note that the scattering amplitude (2) from the spin-spin interaction only involves the combination α_s/m_q^2 .)

The Hamiltonian matrix element h_{fi} is proportional to the Born-order T -matrix element and can be used to calculate Born-order phase shifts through the relation [11]

$$\delta_{Born}^{(\ell)} = -\frac{2\pi^2 k E_1 E_2}{\mathcal{S} E} \int_{-1}^1 h_{fi}(\mu) P_\ell(\mu) d\mu, \quad (3)$$

where E_1 and E_2 are the two hadron energies in the c.m. frame (set equal for $\pi\pi$), E is the total c.m. energy $E_1 + E_2$, and \mathcal{S} is a statistical factor which is 2 for $\pi\pi$ and 1 for $K\pi$. The resulting $I = 0$ $\pi\pi$ phase shifts for even ℓ are

$$\delta_{Born}^{I=0, \ell=\text{even}}(t\text{-ch. gluon}) = +\frac{\alpha_s}{9m_q^2} k E_\pi \left[e^{-x} i_\ell(x) + \frac{8}{\sqrt{27}} e^{-4x/3} \delta_{\ell,0} \right] \quad (4)$$

where $x = k^2/4\beta_\pi^2$. The S-wave phase shift, using $i_0(x) = \sinh(x)/x$, leads to a scattering length of

$$a_0^{I=0}(t\text{-ch. gluon}) = \lim_{k \rightarrow 0} \delta^{0,0}/k = +\frac{1}{9} \left(1 + \frac{8}{\sqrt{27}} \right) \frac{\alpha_s m_\pi}{m_q^2}. \quad (5)$$

This phase shift and scattering length are positive, corresponding to an attractive interaction, but are numerically rather small; with our parameters this scattering length is $a_0^{I=0} = +0.043$ fm, an order of magnitude smaller than the experimental value of

$$a_0^{I=0}(\text{expt.}) = \begin{cases} +0.32(13) \text{ fm} & (\text{production expts}), \\ +0.37(7) \text{ fm} & (K_{e4} \text{ decay}). \end{cases} \quad (6)$$

These numbers are taken from a recent review by Ochs [16], and incorporate constraints from dispersion relations.

This conclusion regarding the small contribution of non-annihilation processes in $I = 0$ $\pi\pi$ is more general than this model, since $a_0^{I=0} = -\frac{1}{2}a_0^{I=2}$ follows from $\pi^+\pi^- \not\rightarrow \pi^+\pi^-$ in any single-pair-interchange model, and one can use the experimental $I = 2$ $\pi\pi$ scattering length of $a_0^{I=2} \approx -0.08$ fm [17] to normalize the no-annihilation amplitude. Evidently S-wave $I = 0$ $\pi\pi$ scattering is dominated by annihilation processes, and the interesting question is whether these are due primarily to a broad f_0 $q\bar{q}$ state (as is often assumed) or to a broad scalar glueball or another intermediate state such as $q\bar{q}g$.

2) *s*-channel gluon exchange

The *s*-channel gluon exchange scattering mechanism is in many ways the most interesting. We will first calculate this contribution using perturbative gluons. If gluons behaved like photons we could model the low-energy effects of *s*-channel gluon exchange by the familiar positronium annihilation interaction [18] augmented by a color factor,

$$H_I^{pert.}(s\text{-ch. gluon}) = +\frac{2\pi\alpha_s}{m_q^2} (\lambda_I^a/2) \cdot (\lambda_F^a/2) (\vec{S}_i \cdot \vec{S}_j + \frac{3}{4}) \delta(\vec{r}_{ij}) . \quad (7)$$

For *s*-channel transverse gluon exchange this is the leading term in an expansion in v^2/c^2 . There is also a color Coulomb interaction, but we expect this to be small because the annihilation color charge density $\langle 0|\rho^a(\vec{x})|q\bar{q}\rangle$ transforms as $L = 1$.

A typical annihilation diagram involving this Hamiltonian, for $K^+\pi^- \rightarrow K^+\pi^-$, is shown in Fig.1. The contribution of this diagram to the meson-meson scattering amplitude can be derived easily using the diagrammatic techniques discussed in [9], see especially Appendix C of that reference. In Ps-Ps scattering this Hamiltonian has a color matrix element of $+4/9$ and a spin matrix element of $+3/4$. We may separate the isospin amplitudes by considering the special cases $\pi^+\pi^- \rightarrow \pi^+\pi^-$ and $\pi^+\pi^- \rightarrow \pi^-\pi^+$ ($\hat{\Omega}_f(\pi^\pm) \rightarrow -\hat{\Omega}_f(\pi^\pm)$); this leads to

$$h_{fi}^{I=0}(s\text{-ch. gluon}) = +\frac{2\pi\alpha_s}{m_q^2} \frac{1}{(2\pi)^3} \left[e^{-(1-\mu)x} + e^{-(1+\mu)x} \right] \quad (8)$$

and

$$h_{fi}^{I=1}(s\text{-ch. gluon}) = +\frac{4\pi\alpha_s}{3m_q^2} \frac{1}{(2\pi)^3} \left[e^{-(1-\mu)x} - e^{-(1+\mu)x} \right] , \quad (9)$$

again using $x = k^2/4\beta_\pi^2$. The $I = 2$ annihilation amplitude is of course zero. Using (3), these amplitudes lead to $I = 0$ and $I = 1$ $\pi\pi$ phase shifts of

$$\delta_{Born}^{I,\ell}(s\text{-ch. gluon}) = c_{I,\ell} \frac{\alpha_s}{m_q^2} k E_\pi e^{-x} i_\ell(x) , \quad (10)$$

where $c_{I,\ell}$ is $-1/2$ for $I = 0$ and $\ell = \text{even}$, $-1/3$ for $I = 1$ and $\ell = \text{odd}$, and zero otherwise. The S-wave phase shift gives an $I = 0$ scattering length of

$$a_0^{I=0}(s\text{-ch. gluon}) = -\frac{1}{2} \frac{\alpha_s m_\pi}{m_q^2} . \quad (11)$$

This negative phase shift and scattering length correspond to a repulsive interaction, opposite to the observed $I = 0$ $\pi\pi$ interaction. With our standard quark model parameters this s -channel massless-gluon contribution is $a_0^{I=0} = -0.076$ fm, somewhat larger than t -channel gluon exchange and opposite in sign.

More careful consideration suggests that the assumption of massless perturbative gluons is especially unphysical in this case and should be modified [19]. One effect of confinement is to raise the mass of the lowest intermediate $q\bar{q}g$ hybrid basis state from the perturbative value of $2m_q$ to the physical hybrid mass M_H , which is well above the $\pi\pi$ invariant masses we consider. This causes the energy denominator $1/(E_{q\bar{q}} - E_g)$ in the $q\bar{q} \rightarrow g \rightarrow q\bar{q}$ subprocess to change sign. Since the sign of the amplitude for this second-order process is also determined by the energy denominator, confinement changes this sign as well, which leads to an attractive force in $I = 0$ $\pi\pi$ and $I = 1/2$ $K\pi$. There are many ways one might incorporate this effect of confinement on virtual gluons; one simple approach is to modify the denominator of the gluon propagator by including an effective gluon mass,

$$s^{-1} \rightarrow (s - \mu_g^2)^{-1}, \quad (12)$$

where the mass scale μ_g is set by the hybrid mass gap, which is presumably about 1 GeV. This modification of the gluon propagator changes the effective low-energy $q\bar{q}$ interaction in (7) by replacing m_q^{-2} by $(m_q^2 - \mu_g^2/4)^{-1}$. This multiplies the naive Hamiltonian (7) by a negative constant,

$$H_I^{conf.}(s\text{-ch. gluon}) = - \left\{ \frac{1}{(\mu_g/2m_q)^2 - 1} \right\} \cdot H_I^{pert.}(s\text{-ch. gluon}). \quad (13)$$

If we estimate constituent masses of $\mu_g = 1$ GeV and $m_q = 0.33$ GeV, we expect the s -channel effective interaction to be similar in magnitude to the original massless gluon form but opposite in sign. The exact numerical strength of this diagram is clearly problematical due to uncertainties in the effect of confinement, so in practice we simply introduce a multiplicative factor f in the s -channel annihilation Hamiltonian and use the form

$$H_I^{conf.}(s\text{-ch. gluon}) = +f \cdot \frac{2\pi\alpha_s}{m_q^2} (\lambda_I^a/2) \cdot (\lambda_F^a/2) (\vec{S}_i \cdot \vec{S}_j + \frac{3}{4}) \delta(\vec{r}_{ij}). \quad (14)$$

The phase shifts (10) and scattering length (11) from s -channel gluon exchange are thus multiplied by a phenomenological parameter f . We will fit f to the experimental $I = 0$ $\pi\pi$ and $I = 1/2$ $K\pi$ S-wave phase shifts, with the caveat that we expect it to be negative and not far from unity.

3) s -channel resonances

We treat the phase shift due to conventional s -channel $q\bar{q}$ resonances phenomenologically, since it is not clear how to determine three-meson couplings directly from quark-gluon interactions. We use a relativized Breit-Wigner form suggested by the Particle Data Group [20] to model the s -channel resonances, which has an elastic phase shift of

$$\delta_R = \tan^{-1} \left\{ \frac{\sqrt{s} \Gamma(E)}{M_R^2 - s} \right\}, \quad (15)$$

where $s = E^2$. Since we are considering broad resonances it is important to include the energy dependence of the width $\Gamma(E)$. Of course this function is somewhat model dependent. In general we expect it to consist of a centrifugal factor for decay to a two-body final state with orbital angular momentum L , times a phase space factor of k/M and a vertex form factor $D(k)$ derived from wavefunction overlaps of the initial and final mesons. Here we use the form suggested by the LASS collaboration [21],

$$\Gamma(E) = \left(\frac{k}{k_R}\right)^{2L} \cdot \left(\frac{k/E}{k_R/M_R}\right) \cdot \left(\frac{D(k)}{D(k_R)}\right) \cdot \Gamma(M_R) . \quad (16)$$

In this formula k_R is the momentum of one final-state hadron at the s -channel resonance mass, and for $\pi\pi$, $k_R = \sqrt{M_R^2/4 - m_\pi^2}$. (Here and in the remainder of the paper we assume single decay channels for each resonance, $\pi\pi$ for f_0 and ρ and $K\pi$ for K_0^* and K^* . The broad scalar resonances couple dominantly to these channels so this is a reasonable first approximation, especially for low-energy effects.)

The vertex form factor $D(k)$ is wavefunction-dependent; here we use

$$D(k) = c \exp \left\{ -k^2/6\beta_\pi^2 \right\} , \quad (17)$$

which follows from the 3P_0 decay model with SHO wavefunctions [22]. We expect the corresponding form factor for radial excitations to suppress the low-energy contributions of radially-excited $q\bar{q}$ states so they are not important numerically.

The low-energy limit of this resonance phase shift gives the contribution of a scalar resonance to the scattering length, which in the $\pi\pi$ (equal mass) case is

$$a_0(\text{res.}) = \frac{1}{(1 - 4m_\pi^2/M_R^2)^{3/2}} \cdot \frac{D(0)}{D(k_R)} \cdot \frac{\Gamma(M_R)}{M_R} \cdot \frac{2}{M_R} . \quad (18)$$

For $I = 0$ $\pi\pi$ scattering the only well established broad resonance is the $f_0(1400)$. The mass and width of this broad state are both rather problematical; the PDG mass and width estimates are $M_R \approx 1400$ MeV and $\Gamma(M_R) \approx 150 - 400$ MeV. For this wide range of values the scattering lengths with the 3P_0 form factor are

$$a_0(\text{res.}) = +(0.06 - 0.17) \text{ fm} . \quad (19)$$

Taking the recent Crystal Ball central values for the mass and width from $\gamma\gamma \rightarrow \pi^0\pi^0$, $M = 1250$ MeV (assumed) and $\Gamma_R = 268(70)$ MeV [23], we find a contribution to the scattering length of

$$a_0(\text{res.}) = +0.11(3) \text{ fm} . \quad (20)$$

The Crystal Barrel collaboration [24] recently reported a similar mass and width for this broad f_0 , $M \approx 1335$ MeV and $\Gamma = 255(40)$ MeV, corresponding to a very similar scattering length of $0.11(2)$ fm. The Crystal Barrel and Obelix [25] collaborations report a similar, somewhat broader f_0 in 4π final states, with $M = 1374(38)$ MeV and $\Gamma = 375(61)$ MeV and $M = 1345(12)$ MeV and $\Gamma = 398(26)$ MeV respectively. These imply a resonance contribution to the scattering length near the upper limit of (19).

Recall for comparison that the nonresonant gluon exchanges give contributions to the scattering length of

$$a_0(\text{gluon ex.}) = \left\{ +0.043 - 0.076f \right\} \text{ fm} , \quad (21)$$

where f is expected to be negative and not large relative to unity. Since the experimental scattering length is about $+(0.3 - 0.4)$ fm (6), these results suggest that low energy S-wave $I = 0$ $\pi\pi$ scattering receives important contributions both from the broad $f_0(1400)$ and from nonresonant scattering, and that $f \approx -2$ to -3 . Our detailed fits to phase shifts will support similar values for f .

The resonance contributions remain large at low energies because of the $1/E$ factor in the energy-dependent width $\Gamma(E)$ (16). Although this appears well motivated as the $1/M$ in the decay rate of an initial state of mass M [26, 27], this factor and the $\delta(\Gamma, E)$ relation (15) are so important to the threshold behavior that they merit more careful study. It would also be interesting to explore the sensitivity of our conclusions to the form chosen for the k -dependent three-meson vertex; using instead a pointlike-meson form factor, we would predict a scattering length from the broad f_0 resonance about half as large. This is because the form factor suppresses the coupling of the f_0 to pions at higher momenta, so if we use the width at resonance as a fixed input, the strength of the coupling at threshold is increased by the form factor. (Note the $D(0)/D(k_R)$ dependence of the scattering length in (18).) Another concern is that scattering amplitudes from two different time-orderings, $\pi\pi \rightarrow f_0 \rightarrow \pi\pi$ and $\pi\pi \rightarrow f_0 \pi\pi \pi\pi \rightarrow \pi\pi$, are added with equal strength to give the covariant form (15). These are actually modified by form factors, and the second “Z-graph” process may be strongly suppressed [28]. We will test the importance of some of these complications in our detailed study of experimental S-wave phase shifts.

3. $I = 1/2$ $K\pi$ Scattering Amplitudes

Application of these techniques to $I = 1/2$ $K\pi$ scattering is straightforward. First, t -channel gluon exchange for $I = 1/2$ is simply related to the $I = 3/2$ amplitude,

$$h_{fi}^{I=1/2}(t\text{-ch. gluon}) = -\frac{1}{2} h_{fi}^{I=3/2}(t\text{-ch. gluon}) , \quad (22)$$

which follows from $K^-\pi^+ \not\leftrightarrow K^-\pi^+$ through this mechanism. Taking the $I = 3/2$ result from [11], we have

$$h_{fi}^{I=1/2}(t\text{-ch. gluon}) = -\frac{1}{(2\pi)^3} \frac{2\pi\alpha_s}{9m_q^2} (T_1 + T_2 + C_1 + C_2) , \quad (23)$$

where T_i and C_i represent the contributions of the “transfer” and “capture” diagrams [9]. Specializing to the case of identical pion and kaon spatial wavefunctions ($\beta_\pi = \beta_K$), these are explicitly

$$T_1 = \exp \left\{ -\frac{(2 + 2\zeta + \zeta^2)(1 - \mu)}{2} x \right\} , \quad (24)$$

$$T_2 = \rho \exp \left\{ -\frac{2 - 2\zeta + \zeta^2 + 2(1 - \zeta)\mu}{2} x \right\} , \quad (25)$$

$$C_1 = \rho (4/3)^{3/2} \exp \left\{ - \frac{4 - \zeta + \zeta^2 - 3\zeta\mu}{3} x \right\} \quad (26)$$

and

$$C_2 = (4/3)^{3/2} \exp \left\{ - \frac{4 + \zeta + 2\zeta^2 - (5\zeta + \zeta^2)\mu}{3} x \right\}, \quad (27)$$

where $x = k^2/4\beta_\pi^2$, as in $\pi\pi$ scattering. These somewhat complicated results allow for different strange and nonstrange quark masses through the parameter

$$\rho = m_q/m_s, \quad (28)$$

and ζ is the combination $(1 - \rho)/(1 + \rho)$. In our previous study of $I = 3/2$ $K\pi$ scattering [11] we considered the more general problem of different pion and kaon length scales, but found that the $I = 3/2$ scattering amplitudes were quite insensitive to the ratio β_π/β_K , and setting it equal to unity allowed a very good description of the data. For equal length scales we found an optimum value of $\rho = 0.677$, which we will also assume here.

The s -channel gluon exchange diagram with $\beta_\pi = \beta_K$ gives a Hamiltonian matrix element of

$$h_{fi}^{I=1/2}(s\text{-ch. gluon}) = +f \cdot \frac{\pi\alpha_s}{m_q^2} \frac{1}{(2\pi)^3} \exp \left\{ - \frac{(2 + 2\zeta + \zeta^2)(1 - \mu)}{2} x \right\}, \quad (29)$$

which by inspection is proportional to the contribution of the t -channel gluon diagram “transfer₁” (T_1 above), and so can be incorporated in (23) by the substitution $T_1 \rightarrow (1 - 9f/2)T_1$. The general- ℓ phase shift and the corresponding scattering length from combined s - and t -channel gluon exchange are

$$\begin{aligned} \delta^{I=1/2,\ell}(\text{gluon ex.}) &= \frac{\alpha_s}{9m_q^2} \frac{kE_\pi E_K}{E} \left[\left(1 - \frac{9}{2}f\right) e^{-(1+\zeta+\frac{1}{2}\zeta^2)x} i_\ell\left(\frac{2+2\zeta+\zeta^2}{2}x\right) \right. \\ &\quad \left. + \rho \left\{ e^{-(2-\zeta+\frac{1}{2}\zeta^2)x} + (4/3)^{3/2} e^{-\frac{1}{3}(4-\zeta+\zeta^2)x} \right\} i_\ell(\zeta x) + (4/3)^{3/2} e^{-\frac{1}{3}(4+\zeta+2\zeta^2)x} i_\ell\left(\frac{5\zeta+\zeta^2}{3}x\right) \right] \end{aligned} \quad (30)$$

and

$$a_0^{I=1/2}(\text{gluon ex.}) = \frac{\alpha_s}{9m_q^2} \frac{m_\pi m_K}{m_\pi + m_K} \left[\left(1 + (4/3)^{3/2}\right)(1 + \rho) - \frac{9}{2}f \right]. \quad (31)$$

The scalar $K_0^*(1430)$ is the single important s -channel resonance in low-energy S -wave $I = 1/2$ $K\pi$ scattering. We incorporate this state using a relativized Breit-Wigner of the form (15-17), as in $\pi\pi$ scattering. It leads to a scattering length of

$$a_0^{I=1/2}(\text{res.}) = \frac{M_R^2}{M_R^2 - (m_\pi + m_K)^2} \frac{M_R}{k_R} \frac{D(0)}{D(k_R)} \frac{\Gamma(M_R)}{M_R} \frac{1}{M_R}. \quad (32)$$

The PDG mass and width for this state (taken from LASS results) are $M = 1429 \pm 4 \pm 5$ MeV and $\Gamma = 287 \pm 10 \pm 21$ MeV [29], but subsequent reanalysis by this group has led to preferred values of $M = 1412$ MeV and $\Gamma = 294$ MeV [26]. These correspond to a contribution to the scattering length of $a_0(K_0^*(1430)) = 0.145$ fm given our form for the relativized Breit Wigner (presumably with a statistical error from Γ of about 10%). In summary, the total scattering

length we expect from gluon exchange and the s -channel resonance $K_0^*(1430)$ is numerically (excluding errors)

$$a_0^{I=1/2}(K\pi \text{ thy.}) = \left\{ 0.056 - 0.059f + 0.145 \right\} \text{ fm} . \quad (33)$$

which we may compare with the experimental scattering length found by Estabrooks *et al.* [30],

$$a_0^{I=1/2}(K\pi \text{ expt.}) = 0.472(8) \text{ fm} . \quad (34)$$

Evidently the scattering lengths (33) and (34) suggest a value of $f \approx -4$ for the s -channel gluon strength parameter, although the individual contributions to the scattering length have important uncertainties so this parameter is not very well determined. In our subsequent analysis of phase shifts we shall find that a somewhat smaller value of $f \approx -2.5$ appears reasonable over a wide range of energies for $\pi\pi$ and $K\pi$ elastic scattering.

4. Comparison to Experimental Phase Shifts

a) $I = 0$ $\pi\pi$ and $I = 1/2$ $K\pi$ S-waves

We begin our detailed comparison with experiment with a study of the $I = 0$ $\pi\pi$ and $I = 1/2$ $K\pi$ S-waves. We combine these because we expect them to be closely related by SU(6) symmetry, and together they provide data on nearly elastic S-wave Ps-Ps scattering amplitudes from invariant masses of ≈ 0.3 to 1.6 GeV.

We impose unitarity on the total scattering amplitude by assuming that the phase shifts for the individual processes add, as they would for small amplitudes;

$$\delta_{tot} = \delta_{Born} + \delta_R . \quad (35)$$

This is equivalent to the prescription used by the LASS collaboration, which was to assume a relative phase between resonant and background amplitudes, chosen so that the complex sum remains on the unitarity circle,

$$\sin(\delta_{tot})e^{i\delta_{tot}} = [\sin(\delta_{bkg})e^{i\delta_{bkg}}] + e^{2i\delta_{bkg}} \cdot [\sin(\delta_R)e^{i\delta_R}] . \quad (36)$$

For these amplitudes the Born phase shift is given by (30) and the $f_0(1400)$ and $K_0^*(1430)$ resonance phase shifts by (15-17). We fix the quark model parameters at $\alpha_s = 0.6$, $m_q = 0.33$ GeV, $\rho = 0.677$ and $\beta_\pi = \beta_K$ (from our previous study of $I = 2$ $\pi\pi$ [9] and $I = 3/2$ $K\pi$ [11] phase shifts). We generally set the pion and kaon masses equal to the isospin averages $m_\pi = 0.138$ GeV and $m_K = 0.495$ GeV, but for the special case $I = 0$ $\pi\pi$ we use $m_\pi = m_{\pi^+} = 0.1396$ GeV, as appropriate for the near-threshold points from K_{e4} decay.

The $I = 0$ $\pi\pi$ channel is the most interesting historically [31], due to long-standing uncertainties in the properties of the very broad f_0 resonance seen in this slowly rising phase shift. This channel is further complicated by inelasticities associated with the narrow $f_0(975)$, and a scalar glueball is expected near 1.5 GeV, so in this single channel study we consider only invariant masses below 0.9 GeV. For our $I = 0$ $\pi\pi$ data set we use the s -channel and t -channel extrapolations of Estabrooks and Martin [32] (38 points from 0.51

GeV to 0.89 GeV), the “case 1” phase shift of Protopopescu *et al.* [33] (17 points from 0.55 GeV to 0.89 GeV), and the low-energy data from K_{e4} decay of Rosselet *et al.* [34] (5 points for $\delta^{I=0,\ell=0} - \delta^{I=1,\ell=1}$ from 0.289 GeV to 0.367 GeV). We added an estimated low-energy P-wave phase shift proportional to k_π^3 to the Rosselet data, with a coefficient chosen to give 9.4° at 0.51 GeV, as reported by Estabrooks and Martin [32].

For our $K\pi$ data set we use the 24 data points of Estabrooks *et al.* [30] (from 0.73 to 1.30 GeV, the full set of separated-isospin phases quoted) and the 37 data points of Aston *et al.* [21] below 1.6 GeV. This cutoff was chosen because the radial $K_0^*(1950)$ should become important above this mass. Since the LASS collaboration [21] tabulated only the $K^-\pi^+$ phase shift ϕ [35] rather than the separated isospin ones, we used our formalism to calculate $I = 3/2$ S-wave shifts as well to fit this ϕ data directly. We will show the results of the fit for $I = 1/2$ phase shifts, and the experimental errors we show in the figure are actually those quoted by LASS for the mixed-isospin phase ϕ . This gives a total data set of 121 points for S-wave Ps-Ps scattering, from an invariant mass of 0.289 GeV in $\pi\pi$ to 1.59 GeV in $K\pi$.

We fitted this full S-wave data set with the single s -channel annihilation strength parameter f and masses and widths for the two broad resonances $f_0(1400)$ and $K_0^*(1430)$. Since the mass of the $f_0(1400)$ is poorly determined due to our 0.9 GeV cutoff in the $\pi\pi$ data, we constrained the masses by $M_{f_0} = M_{K_0^*} - 0.12$ GeV, as suggested by the masses of other members of the 3P_J SU(3) flavor multiplet. There is some indication of an $f_0(975)$ contribution to the phase shift near 0.9 GeV, so we also added a conventional elastic Breit-Wigner phase shift with $M = 0.974$ MeV and $\Gamma = 0.047$ GeV (PDG values, both fixed) to the $I = 0$ $\pi\pi$ phase shift. The optimum values of the parameters were found to be $f = -2.573$, $M_{K_0^*} = 1.477$ GeV, $\Gamma_{K_0^*} = 0.261$ GeV, $M_{f_0} = 1.357$ GeV (constrained by $M_{K_0^*}$), and $\Gamma_{f_0} = 0.405$ GeV. The experimental and fitted phase shifts are shown in Fig.2 ($K\pi$) and Fig.3 ($\pi\pi$), and except for some inaccuracies in describing the shape of the $K_0^*(1430)$ resonance region these four parameters evidently give a reasonable description of both amplitudes.

The resonance widths found in this fit are especially attractive; $\Gamma_{f_0} = 0.405$ GeV is reasonably consistent with the recent experimental values of 0.255(40) [24], 0.268(70) [23], 0.375(61) [24], and 0.398(26) GeV [25]. The discrepancy between these moderate widths (from production of the f_0 in $\gamma\gamma$ and $P\bar{P}$) and the very large widths seen in scattering can thus be understood as due to the additional nonresonant contributions to scattering, which make the f_0 and K_0^* appear broader than they actually are. It is also reassuring that the relative fitted f_0 and K_0^* widths are not far from the naive SU(6) expectation of $\Gamma_{f_0}/\Gamma_{K_0^*} = 2$ (assuming only $\pi\pi$ and $K\pi$ modes and neglecting phase space differences).

The fitted mass for the broad f_0 , $M_{f_0} = 1.357$ GeV, compares well with experimental values from recent $P\bar{P}$ annihilation experiments, 1.335(30) [24], 1.345(12) [25], and 1.374(38) GeV [24]. Our fitted K_0^* mass of 1.477 GeV is significantly higher than the LASS value of 1.412 GeV, although an independent analysis of the LASS data by Weinstein and Isgur [6] using a two-channel Schrödinger formalism found 1.47 GeV, essentially identical to our result. The discrepancy appears to be due mainly to the use by LASS of the phase shift formula (15) with \sqrt{s} replaced by M_R ; on fitting our $K\pi$ S-wave data set with their form we find $M_{K_0^*} = 1.432$ GeV. Although the most reasonable generalization of the Breit-Wigner form to a broad resonance is not well established theoretically, we found that (15) with \sqrt{s} replaced by M_R gives an unsatisfactory fit to the low energy $I = 0$ $\pi\pi$ data (a shoulder is predicted near threshold), and for this reason we have used the PDG form (15).

The scattering lengths in this fit and the quark Born diagram (nonresonant) and resonance contributions are as follows:

$$a_0^{I=0}(\pi\pi) = 0.41 \text{ fm} = 0.24 \text{ fm(nonres.)} + 0.17 \text{ fm(res.)} , \quad (37)$$

$$a_0^{I=1/2}(K\pi) = 0.33 \text{ fm} = 0.21 \text{ fm(nonres.)} + 0.12 \text{ fm(res.)} . \quad (38)$$

Although Pennington and especially Burkhardt and Lowe [16] prefer a value closer to 0.3 fm for the $I = 0$ $\pi\pi$ scattering length [16], it is clear from Fig.3 that our fit (with 0.41 fm) gives a satisfactory description of the existing low-energy $I = 0$ $\pi\pi$ data, and considerably improved low energy measurements will be required to distinguish these values. The larger value of about 0.472(8) fm cited by Estabrooks *et al.* [30] for the $I = 1/2$ $K\pi$ scattering length may give a somewhat improved fit to the low-energy $K\pi$ data (see Fig.2), but a better measurement of S-wave phase shifts near and below 0.9 GeV would be required to confirm this larger value. If improvements in the low energy data do confirm a value of 0.3 fm for $I = 0$ $\pi\pi$, this could be accommodated in our formalism by reducing the broad f_0 width to about 300 MeV and changing f to -2 . It is difficult however to see how a $K\pi$ scattering length of 0.47 fm could be fitted simultaneously without unrealistic parameter changes. For this reason we suggest that the error in (34) was underestimated, and that a value near 0.3 fm will be found in a higher-statistics measurement of low energy $I = 1/2$ $K\pi$ scattering.

To test the stability of the fitted parameter values we performed fits with f fixed and the resonance masses and widths free (but with $M_{K_0^*} - M_{f_0} = 0.12$ GeV constrained). The variation of the fit residual $F = \kappa \sum_i (\delta_i(\text{expt.}) - \delta_i(\text{thy.}))^2 / \epsilon_i(\text{expt.})^2$ and the resonance parameters with f is shown in Fig.4. (We normalize F to unity for $f = 0$.) Evidently nonzero f improves the fit considerably, by better than a factor of two relative to $f = 0$. There is a strong correlation between f (which provides a smoothly rising phase shift) and the resonance widths; with $f = 0$ the best fit requires implausibly large widths of $\Gamma_{f_0} = 1.47$ GeV and $\Gamma_{K_0^*} = 0.57$ GeV. As the nonresonant scattering is increased with $-f$, the widths of the resonances required to describe the balance of the low-energy scattering fall. By $f = -2$ the widths have fallen to the more reasonable values $\Gamma_{f_0} = 0.61$ GeV and $\Gamma_{K_0^*} = 0.34$ GeV. There is a region of comparable quality of fit, $-2 > f > -3$, although by $f = -3$ the fitted f_0 width is 0.27 GeV, near the lowest experimental estimates.

Finally, we tested some of the uncertainties in modelling broad resonances discussed in Sec.2.3 by carrying out fits to the $K\pi$ data set alone with different forms for the resonance. The PDG form (15) gives $(M_{K_0^*}(\text{GeV}), \Gamma_{K_0^*}(\text{GeV}), f) = (1.477, 0.257, -2.602)$; the LASS form (as discussed above) gives $(1.432, 0.308, -1.403)$; the LASS form with a single time ordering (which is a nonrelativistic Breit-Wigner with the energy-dependent width (16)) gives $(1.457, 0.284, -2.011)$; the LASS form with no meson form factor gives $(1.477, 0.268, -2.584)$; and finally, a simple nonrelativistic Breit-Wigner phase shift with no meson form factor and a constant width gives $(1.471, 0.290, -2.393)$. Evidently the various assumptions about the resonance phase shift lead to variations of about ± 20 MeV in the mass and width and about ± 0.5 in the parameter f .

b) $I = 1$ $\pi\pi$ and $I = 1/2$ $K\pi$ P-waves

The higher partial waves may be treated similarly, and there are fewer complications because the resonances are narrower and the Born contributions are smaller. Indeed, a

casual inspection of the $I = 1 \pi\pi$ and $I = 1/2 K\pi$ P-wave phase shift data to 1.5 GeV shows clear evidence for the $\rho(770)$ and $K^*(892)$ and little else.

For our $I = 1 \pi\pi$ P-wave data set we again use the s -channel and t -channel extrapolations of Estabrooks and Martin [32] (42 points from 0.51 GeV to 0.97 GeV) and the “case 1” phase shift of Protopopescu *et al.* [33] (26 points from 0.55 GeV to 1.15 GeV). For $K\pi$ we choose to fit only the Estabrooks *et al.* [30] P-wave data (24 points, at energies described above); since the $I = 3/2$ P-wave is not well established we cannot use the mixed-isospin P-wave phase shift ϕ tabulated in the LASS data [35].

To fit these P-waves we again use the resonant phase shift (15-17) and set $\ell = 1$ in (30). The quark model parameters α_s , m_q , m_s and $\beta_\pi = \beta_K$ and the meson masses m_π and m_K were assigned the same values as in the S-wave fit. Since the quark Born diagram contributions to the P-wave phase shifts are rather small (typically about 5° at 1 GeV invariant mass), we do not fit them independently but instead assume the value of f found in the S-wave scattering amplitudes, $f = -2.573$. This leaves the ρ and K^* masses and widths as free parameters. Fitting the 92 P-wave phase shift data points gives $M_\rho = 0.7703$ GeV, $\Gamma_\rho = 0.1563$ GeV, $M_{K^*} = 0.8950$ GeV and $\Gamma_{K^*} = 0.0544$ GeV; these are in quite good agreement with PDG values, the largest discrepancies being about 5 MeV in both widths. Note that the nonresonant amplitudes play an important part in bringing about this close agreement; imposing $f = 0$ makes the fit less than half as accurate (it more than doubles F) and leads to ρ parameters of $M_\rho = 0.7633$ GeV and $\Gamma_\rho = 0.1434$ GeV, which do not reproduce the PDG values of $M_\rho = 0.7681(5)$ GeV and $\Gamma_\rho = 0.1515(12)$ GeV so well.

The general features of the $\pi\pi$ and $K\pi$ fits are quite similar, so we display results from $K\pi$ only, which has smaller errors at the higher energies. The fit to the $I = 1/2 K\pi$ P-wave is shown in Fig.5, which also shows the individual contributions from the $K^*(892)$ and the s - and t -channel Born diagrams. Evidently the quark Born diagram contribution is dominated by s -channel gluon exchange (hybrid diagrams). The t -channel diagrams give zero for $I = 1 \pi\pi$ scattering since they are even under $\theta \rightarrow -\theta$. The $K\pi$ P-wave is related to the $\pi\pi$ P-wave by SU(6) symmetry, and the $K\pi$ t -channel contribution is nonzero only because of SU(6) violation through $m_s \neq m_{u,d}$. It appears that the resonance shape alone does not give a good description of the phase shift somewhat above the K^* (note especially the 0.95 to 1.25 GeV region, and the Born diagrams (with no free parameters) give just about the contribution required for good agreement with experiment. This may be correct, but we must be cautious in this interpretation; the smaller resonance contribution is due to the energy-dependent width $\Gamma(E)$ in (15), which is increasing quite rapidly for the K^* in this mass region. A similar effect is seen in the $I = 1 \pi\pi$ fit. Since this $\Gamma(E)$ is rather uncertain, we can only conclude that the quark Born diagrams give a contribution to P-wave phase shifts which is consistent with data, and may be observable somewhat above the ρ and K^* masses. The systematic uncertainties in describing the resonance phase shifts however are comparable to the Born diagram contributions, and until the resonance contributions can be established with better accuracy we cannot claim to have confirmed the predicted quark Born amplitudes in this P-wave data.

5. Conclusion and Suggestions for Future Studies

We have extended the constituent interchange model of Barnes and Swanson to processes with valence $q\bar{q}$ annihilation by incorporating s -channel gluon exchange and s -channel relativized Breit-Wigner resonances. We applied these techniques to $I = 0$ and $I = 1$ $\pi\pi$ and $I = 1/2$ $K\pi$ scattering in S- and P-waves, since these amplitudes are well established experimentally and can be studied for evidence of nonresonant scattering in addition to the known s -channel resonances. In a simultaneous fit to the $\pi\pi$ and $K\pi$ S-waves we determined the strength of the s -channel gluon exchange (hybrid) diagram and fitted the masses and widths of the $K^*(1430)$ and the problematical $f_0(1400)$. In our best fit we find an f_0 mass and width of 1357 MeV and 405 MeV, comparable to recent results from $\gamma\gamma$ and $P\bar{P}$ production experiments. We conclude that most of the low energy S-wave scattering in these channels is due to nonresonant s -channel gluon annihilation, which makes the resonances appear very broad in phase shift data. We also applied these techniques to the $I = 1$ $\pi\pi$ and $I = 1/2$ $K\pi$ P-wave phase shifts, and concluded that the s -channel resonances $\rho(770)$ and $K^*(892)$ alone suffice to describe most of the P-wave phase shifts from threshold to over 1 GeV. There may be evidence for the rather small contribution expected from the quark Born diagrams in the P-waves near and above 1 GeV.

We repeatedly found that uncertainties in the generalization of the nonrelativistic Breit-Wigner phase shift to broad resonances limited our ability to separate the resonant and nonresonant contributions to scattering in these channels. For this reason it would be very useful to establish the limitations and range of validity of the PDG form (15) we have assumed in most of this work.

These techniques could be applied most usefully to reactions in which nonresonant s -channel gluon exchange dominates s -channel $q\bar{q}$ resonances. One such possibility is $P\bar{P} \rightarrow \Lambda\bar{\Lambda}$ [36], which has been the subject of detailed experimental investigation at LEAR. It would be interesting to determine whether the s -channel gluon Hamiltonian (14), with strength f fitted to Ps-Ps elastic scattering, also gives an accurate description of this $q\bar{q} \rightarrow s\bar{s}$ annihilation process. This study would presumably require incorporation of initial- and final-state interactions in a coupled channel formalism. This reaction has already attracted considerable theoretical interest, and analyses using similar techniques have been reported [37].

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References

- [1] See for example D.A.Lieberman, Phys. Rev. D16, 1542 (1977); M.Oka and K.Yazaki, Phys. Lett. 90B, 41 (1980); A.Fäbber, F.Fernandez, G.Lübeck and K.Shimizu, Nucl. Phys. A402, 555 (1983); Y.Suzuki and K.T.Hecht, Phys. Rev. C27, 299 (1983).
- [2] K.Maltman and N.Isgur, Phys. Lett. 50, 1827 (1983).
- [3] K.Shimizu, Rep. Prog. Phys. 52, 1 (1989).
- [4] J.Weinstein and N.Isgur, Phys. Rev. Lett. 48, 659 (1982); Phys. Rev. D27, 588 (1983).
- [5] J.Weinstein and N.Isgur, Phys. Rev. D41, 2236 (1990).
- [6] J.Weinstein, Phys. Rev. D43, 95 (1991).
- [7] I.Bender and H.G.Dosch, Z.Phys. C13, 69 (1982); I.Bender, H.G.Dosch, H.J.Pirner and H.G.Kruse, Nucl. Phys. A414, 359 (1984); R.K.Campbell and D.Robson, Phys. Rev. D36, 2682 (1987); H.J.Pirner and D.Povh, Phys. Lett. 114B, 308 (1982); D.Mukhopadhyay and H.J.Pirner, Nucl. Phys. A442, 605 (1985).
- [8] M.Agnello *et al.*, Frascati report LNF-93-021-IR (May 1993).
- [9] T.Barnes and E.S.Swanson, Phys. Rev. D46, 131 (1992).
- [10] E.S.Swanson, Ann. Phys. (NY) 220, 73 (1992); for closely related recent work see also B.Masud, J.Paton, A.M.Green and G.Q.Lin, Nucl. Phys. A528, 477 (1991); D.Blaschke and G.Röpke, Phys. Lett. B299, 332 (1993); B.Masud, “A Model for $q^2\bar{q}^2$ Systems, Illustrated by an Application to $K\bar{K}$ Scattering”, University of Helsinki report (October 1993).
- [11] T.Barnes, E.S.Swanson and J.Weinstein, Phys. Rev. D46, 4868 (1992).
- [12] K.Dooley, E.S.Swanson and T.Barnes, Phys. Lett. 275B, 478 (1992).
- [13] T.Barnes and E.S.Swanson, MIT/ORNL report MIT-CTP-2169/ORNL-CCIP-92-15, to appear in Phys. Rev. C.
- [14] T.Barnes, S.Capstick, M.D.Kovarick and E.S.Swanson, Phys. Rev. C48, 539 (1993).
- [15] N.Isgur, Acta Physica Austriaca, Suppl. XXVII, 177 (1985); P.J.Siemens and A.S.Jensen, *Elements of Nuclei; Many-Body Physics with the Strong Interaction*, Sec. 2.6 (Addison-Wesley, 1987).
- [16] W.Ochs, “Results on Low Energy $\pi\pi$ Scattering”, Max-Planck-Institut für Physik report MPI-Ph/Ph 91-35 (June 1991). In [17] Pennington quotes a similar value of 0.29(6) fm, and Burkhardt and Lowe (using additional chiral symmetry arguments) report 0.282(14) fm.

- [17] This number is taken from our discussion in [9], and is actually not yet very well determined. Several similar values have been reported using various assumptions to extract $\pi\pi$ elastic amplitudes, see for example M.R.Pennington, pp.379-418, DAPHNE Physics Handbook (eds. L.Maiani, G.Pancheri and N.Paver), and Frascati report LNF-92/085 (November 1992), $a_0^{I=2} = -0.086(21)$ fm; M.E.Seior *et al.*, Phys. Rev. D48, 3987 (1993), $a_0^{I=2} = -0.057(4)$ fm; H.Burkhardt and J.Lowe, Phys. Rev. Lett. 67, 2622 (1991), $a_0^{I=2} = -0.046(6)$ fm.
- [18] C.Itzykson and J.Zuber, *Quantum Field Theory* (McGraw-Hill, 1980), pp.484.
- [19] We are indebted to N.Isgur and S.R.Sharpe for discussions of this point.
- [20] Particle Data Group, Phys. Rev. D45, S1 (1992) (final form discussed on p.III.51).
- [21] D.Aston *et al.*, Nucl. Phys. B296, 493 (1988).
- [22] A.LeYaouanc *et al.*, *Hadron Transitions in the Quark Model* (Gordon and Breach, New York, 1988), pp.105.
- [23] J.H.Bienlein, in Proceedings of the IXth International Workshop on Photon-Photon Collisions (San Diego, 22-26 March 1992), eds. D.O.Caldwell and H.P.Paar (World Scientific, 1992), pp. 241-257.
- [24] D.V.Bugg, "Hadron Spectroscopy", in Proceedings of the International Europhysics Conference on High Energy Physics (Marseille, 22-28 July 1993).
- [25] A.Lanaro *et al.*, Nucl. Phys. A558, 13c (1993).
- [26] W.Dunwoodie, personal communication.
- [27] J.D.Bjorken and S.Drell, *Relativistic Quantum Fields* (McGraw-Hill, 1964), Appendix B.
- [28] N.Isgur, C.Morningstar and C.Reader, Phys. Rev. D39, 1357 (1989).
- [29] Particle Data Group, Phys. Rev. D45, S1 (1992).
- [30] P.Estabrooks, R.K.Carnegie, A.D.Martin, W.M.Dunwoodie, T.A.Lasinski and D.W.G.S.Leith, Nucl. Phys. B133, 490 (1978).
- [31] See for example D.Morgan, Phys. Lett. 51B, 71 (1974); N.A.Tornqvist, Phys. Rev. Lett. 49, 624 (1982); Particle Data Group, Rev. Mod. Phys. 56, pp.S159-S162 (1984).
- [32] P.Estabrooks and A.D.Martin, Nucl. Phys. B79, 301 (1974).
- [33] S.D.Protopopescu *et al.*, Phys. Rev. D7, 1279 (1973).
- [34] L.Rosselet *et al.*, Phys. Rev. D15, 1279 (1977).
- [35] N.Awaji, Nagoya University Ph.D. thesis (1986), Appendix B.

- [36] P.D.Barnes *et al.*, Nucl. Phys. A526, 575 (1991), and references cited therein.
- [37] See for example M.Kohno and W.Weise, Nucl. Phys. A479, 433c (1988), and references cited therein.

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